**1.6.1**. The rotationhas complex eigenvalues:

and .

Check thatequals the trace of(sumdown the diagonal). Check thatequals the determinant. Check that those complex eigenvectors are orthogonal, using the complex dot product(not just!). What isand what are its eigenvalues?

**Sol**.

, thus orthogonal.

**1.6.2**. Compute the eigenvalues and eigenvectors ofand. Check the trace! and . has the \_\_\_\_\_ eigenvectors as. Whenhas eigenvaluesand, its inverse has eigenvalues \_\_\_\_\_ .

**Sol**. with & with

with & with

has the same eigenvectors as. Whenhas eigenvaluesand, its inverse has eigenvaluesand.

**1.6.3**. Find the eigenvalues ofand(easy for triangular matrices) and: andand. Eigenvalues of(*are equal to*)(*are not equal to*) eigenvalues ofplus eigenvalues of.

**Sol**.

Eigenvalues ofare not equal to eigenvalues ofplus eigenvalues of.

**1.6.4**. Find the eigenvalues ofandandand:andandand.

(a) Are the eigenvalues ofequal to eigenvalues oftimes eigenvalues of?

(b) Are the eigenvalues ofequal to the eigenvalues of?

**Sol**. (a) The eigenvaluesofare not the product of the eigenvaluesofand the eigenvaluesof.

(b) The eigenvaluesofare the same of the eigenvaluesof.

**1.6.5**. (a) If you know thatis an eigenvector, the way to findis to \_\_\_\_\_.

(b) If you know thatis an eigenvalue, the way to findis to \_\_\_\_\_.

**Sol**. (a) If you know thatis an eigenvector, the way to findis to seeis how many times of.

(b) If you know thatis an eigenvalue, the way to findis to find the solution of.

**1.6.6**. Find the eigenvalues and eigenvectors for both of these Markov matricesand. Explain from those answers whyis close to:

and

**Sol**.

where

**1.6.7**. The determinant ofequals the product. Start with the polynomialseparated into itsfactors (always possible). Then set: so \_\_\_\_\_. Check this rule in Example 4 where the Markov matrix hasand.

**Sol**.

**1.6.8**. The sum of the diagonal entries (the trace) equals the sum of the eigenvalues: has. The quadratic formula gives the eigenvaluesand\_\_\_. Their sum is \_\_\_. Ifhasandthen\_\_\_\_\_.

**Sol**. , , .

**1.6.9**. Ifhasandthen. Find three matrices that have traceand determinantand,.

**Sol**.

**1.6.10**. Choose the last rows ofandto give eigenvalues,and,,: Companion matrices

**Sol**.

**1.6.11**. The eigenvalues ofequal the eigenvalues of. This is because. That is true because \_\_\_\_\_. Show by an example that the eigenvectors ofandare not the same.

**Sol**. is true because

but

**1.6.12**. This matrix is singular with rank one. Find three's and three eigenvectors:

**Sol**. , , , , .

**1.6.13**. Supposeandhave the same eigenvalues,,with the same independent eigenvectors,,. Then. Reason: Any vectoris a combination. What is? What is?

**Sol**.

**1.6.14**. Supposehas eigenvalueswith independent eigenvectors.

(a) Give a basis for the nullspace and a basis for the column space.

(b) Find a particular solution to. Find all solutions.

(c)has no solution. If it did then \_\_\_\_\_ would be in the column space.

**Sol**. (a) Column basis:.

Nullspace:

(b) , whereis a free variable.

(c) has no solution. If it did thenwould be in the column space.

**1.6.15**. (a) Factor these two matrices into: and.

(b) If, thenand

**Sol**. (a) and

(b) If, thenand

**1.6.16**. Suppose. What is the eigenvalue matrix for? What is the eigenvector matrix? Check that

**Sol**. so the eigenvalues become

**1.6.17**. True or false: If the columns of(eigenvectors of) are linearly independent, then

(a) is invertible (b) is diagonalizable (c) is invertible (d) is diagonalizable.

**Sol**. (a) is false. (b) is true. (c) is true. (d) is false.

**1.6.18**. Write down the most general matrix that has eigenvectorsand.

**Sol**.

**1.6.19**. True or false: If the eigenvalues ofare 2, 2, 5, then the matrix is certainly

(a) invertible (b) diagonalizable (c) not diagonalizable

**Sol**. (a) is true. (b) is false. (c) is false.

**1.6.20**. True or false: If the only eigenvectors ofare multiples ofthenhas

(a) no inverse (b) a repeated eigenvalue (c) no diagonalization.

**Sol**. (a) is false. (b) is true. (c) is true.

**1.6.21**. approaches the zero matrix asif and only if everyhas absolute value less than \_\_\_.

Which of these matrices has? and

**Sol**. approaches the zero matrix asif and only if everyhas absolute value less than 1.

**1.6.22**. Diagonalizeand computeto prove this formula for: has.

**Sol**.

**1.6.23**. The eigenvalues ofareand, and the eigenvalues ofareand: and .

Find a matrix square root offrom. Why is there no real matrix square root of?

**Sol**.

**1.6.24**. Suppose the samediagonalizes bothand. They have the same eigenvectors inand.

Prove that.

**Sol**.

**1.6.25**. The transpose ofis. The eigenvectors inare the columns of that matrix. They are often called left eigenvectors of, because. How do you multiply matrices to find this formula for?

Sum of rank-1 matrices

**Sol**.

**1.6.26**. When is a matrix similar to its eigenvalue matrix ? and always have the same eigenvalues. But similarity requires a matrix with . Then is the \_\_\_\_\_ matrix and must have independent \_\_\_\_\_ .

**Sol**. Columns of is the eigenvectors of . So is the eigenvector matrix. is invertible, so its columns are independent, and thusmust haveindependent eigenvectors.